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$B \rightarrow K\eta^{(\prime)}$ decay in perturbative QCD

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Abstract

We compute $B \rightarrow K\eta^{(\prime)}$ branching ratio using perturbative QCD approach. We show that a triangular relation among amplitudes for $B^0 \rightarrow K^0\pi^0$, $B^0 \rightarrow K^0\eta$, $B^0 \rightarrow K^0\eta'$ receives large corrections from $SU(3)$ breaking effects. If experimental value will come closer to the lower limit of the present BELLE data there will be a possibility to understand the large branching ratio of $B^0 \rightarrow K^0\eta'$. Otherwise, we perhaps need to modify our understanding of η' meson, for example, inclusion of a possible admixture of gluonium state.

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Three years has passed since CLEO announced an unexpectedly large branching ratio for $B \rightarrow K\eta'$ decays [1]:

$$Br(\bar{B}^0 \rightarrow \bar{K}^0\eta') = (89_{-16}^{+18} \pm 9) \times 10^{-6} \quad (1)$$

BELLE also reported their results in $BCP4$ conference, [2]:

$$Br(\bar{B}^0 \rightarrow \bar{K}^0\eta') = (64_{-20}^{+25+10}) \times 10^{-6} \quad (2)$$

Various theoretical suggestions have been made to understand the large branching ratio. While new physics contributions were discussed [3, 4] we feel that better understanding of the standard model calculation of the branching ratio is necessary. In Ref. [5], it was shown that there is a possible choice of theoretical parameters involving form factors, CKM parameters, nonfactorizable contribution and decay constants of $\eta - \eta'$ system which gives a branching ratio consistent with the experimental data. A $SU(3)$ relation which is independent of most of the above mentioned uncertainties has been derived [6, 7, 8]:

$$-3\sqrt{2}A(B^0 \rightarrow K^0\pi^0) + 4\sqrt{3}A(B^0 \rightarrow K^0\eta) = \sqrt{6}A(B^0 \rightarrow K^0\eta'). \quad (3)$$

Using the CLEO measurement [9], $Br(\bar{B}^0 \rightarrow \bar{K}^0\pi^0) = (14.6_{-5.1-3.3}^{+5.9+2.4}) \times 10^{-6}$, and theoretical expectation that $|A(B^0 \rightarrow K^0\eta)|$ is small compared to the other two amplitudes, the observed value for $Br(\bar{B}^0 \rightarrow \bar{K}^0\eta')$ in Eq. (1) seems to be too large. This relation also excludes explanations which make $Br(\bar{B}^0 \rightarrow \bar{K}^0\pi^0)$ increase simultaneously with $Br(\bar{B}^0 \rightarrow \bar{K}^0\eta')$, for instance, invoking large Wilson coefficients with new physics effects, or increasing the input parameters like form factors, the CKM parameters, etc.

In this letter, we perform calculation of the branching ratio by using perturbative QCD (pQCD) approach and examine the $B \rightarrow K\eta'$ problem. The $SU(3)$ breaking effect is included through the decay constants and the wave function and as a result, Eq. (3) is modified. We also give a theoretical estimate of color suppressed penguin contributions which is one of the candidate mechanism to enhance $Br(B^0 \rightarrow K^0\eta^{(\prime)})$, but not $Br(\bar{B}^0 \rightarrow \bar{K}^0\pi^0)$.

In the 80's, η' gluonic admixture was examined in Ref. [10]. Recently, there have been some progress in understanding the $\eta - \eta'$ system. We use the $\eta - \eta'$ mixing angle and the definition of the decay constant in $\eta - \eta'$ system which include recent improvements. A simple description of $\eta - \eta'$ states was introduced in the literature,

$$|\eta\rangle = X_\eta \left| \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \right\rangle + Y_\eta |s\bar{s}\rangle \quad (4)$$

$$|\eta'\rangle = X_{\eta'} \left| \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \right\rangle + Y_{\eta'} |s\bar{s}\rangle + Z_{\eta'} |gluonium\rangle, \quad (5)$$

where $X_{\eta^{(\prime)}}$, $Y_{\eta^{(\prime)}}$ and $Z_{\eta^{(\prime)}}$ parameters represent the ratios of $u\bar{u} + d\bar{d}$, $s\bar{s}$ and gluonium component of $\eta^{(\prime)}$, respectively. This work was updated by one of the authors [11]. In this work, the gluonium content of η' is reanalyzed using all available radiative light meson decays and a result

$$\frac{Z_{\eta'}}{X_{\eta'} + Y_{\eta'} + Z_{\eta'}} \leq 0.26 \quad (6)$$

is obtained, which indicates that 26% of gluonic admixture in η' is still possible.

Ignoring the small tree contribution, we can write the amplitudes of $B^0 \rightarrow K^0\pi^0$ and $B^0 \rightarrow K^0\eta^{(\prime)}$ decays as:

$$A(B^0 \rightarrow K^0\pi) = -1/\sqrt{2}P_d \quad (7)$$

$$A(B^0 \rightarrow K^0\eta) = X_\eta/\sqrt{2}P_d + Y_\eta P_s + P \quad (8)$$

$$A(B^0 \rightarrow K^0\eta') = X_{\eta'}/\sqrt{2}P_d + Y_{\eta'} P_s + P', \quad (9)$$

where $P_{d(s)}$ includes color allowed $bsd\bar{d}(s\bar{s})$ penguin and annihilation penguin contributions and $P^{(\prime)}$ is $SU(3)$ singlet contribution. We depict the corresponding diagrams in Fig.1.

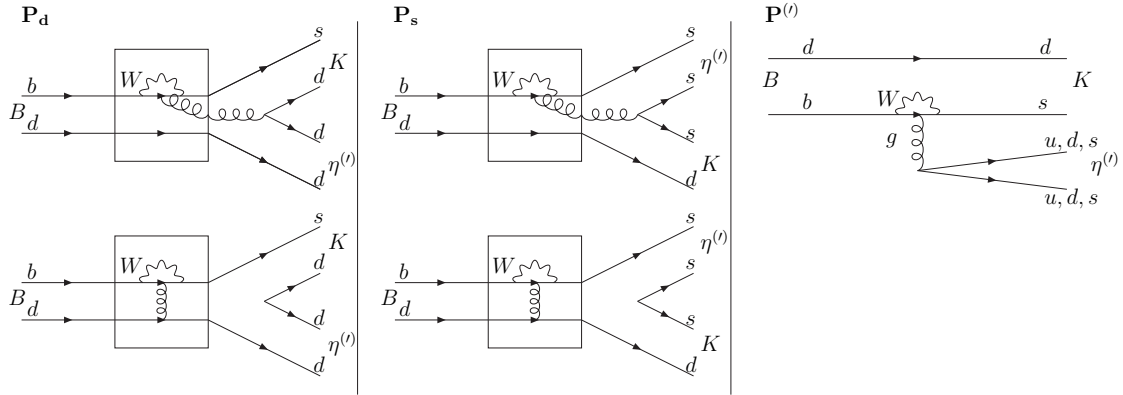


Figure 1: Diagrams for $bsd\bar{d}$ penguin, P_d , $bss\bar{s}$ penguin, P_s and $SU(3)$ singlet penguin, P' contributions

Here we introduce a parameter r which represents the $SU(3)$ breaking effect:

$$P_s = rP_d \quad (10)$$

and a parameter $s^{(\prime)}$ which represents the ratio between $P^{(\prime)}$ and P_s :

$$P = sP_s, \quad P' = s'P_s. \quad (11)$$

Using these parameters, Eqs. (7-9) lead to $SU(3)$ relation in general form

$$A(B^0 \rightarrow K^0 \eta) = -(X_\eta + r\sqrt{2}Y_\eta + r\sqrt{2}s)A(B^0 \rightarrow K^0 \pi^0) \quad (12)$$

$$A(B^0 \rightarrow K^0 \eta') = -(X_{\eta'} + r\sqrt{2}Y_{\eta'} + r\sqrt{2}s')A(B^0 \rightarrow K^0 \pi^0). \quad (13)$$

Now, let us examine what it takes to obtain Eq. (3). The following assumptions must be applied:

- η' does not have the gluonic content ($Z_{\eta'} = 0$) so that $X_{\eta(\eta')}$ and $Y_{\eta(\eta')}$ are related to the pseudoscalar mixing angles as

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \alpha_p & -\sin \alpha_p \\ \sin \alpha_p & \cos \alpha_p \end{pmatrix} \begin{pmatrix} \frac{u\bar{u}+d\bar{d}}{\sqrt{2}} \\ s\bar{s} \end{pmatrix}. \quad (14)$$

For convenience, we also display η and η' states in terms of singlet state η_1 and octet state η_8 as

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta_p & -\sin \theta_p \\ \sin \theta_p & \cos \theta_p \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}, \quad (15)$$

where $\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$ and $\eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$. α_p can be written in terms of the pseudoscalar mixing angle θ_p as $\alpha_p = \theta_p - \theta_I + \frac{\pi}{2}$ with the ideal mixing $\theta_I = \arctan(1/\sqrt{2})$. The allowed range of values for θ_p is -20° to -10° . To obtain Eq. (3), the angle α_p is fixed at $\cos \alpha_p = \sqrt{2}/\sqrt{3}$ and $\sin \alpha_p = 1/\sqrt{3}$, which corresponds to $\theta_p \approx -19.4^\circ$. We should remind the reader that we are taking a value of θ_p which is at the edge of the allowed region. While we stretched the error bar to allow the region $\theta_p \sim -20^\circ$, the experimental data for $\omega \rightarrow \eta\gamma$ decay and $\eta \rightarrow \gamma\gamma$ decay disfavor this region. The best fit range is: -17° to -10° in [11].

- $SU(3)$ symmetry is exact so that $P_d = P_s$, *i.e.* $r = 1$.
- The ratio of the $SU(3)$ flavor singlet contribution to $B \rightarrow K\eta$ and $B \rightarrow K\eta'$ is written as $s'/s = -\cos \theta_p / \sin \theta_p$, which is extracted from the ratio of the $SU(3)$ singlet component of η and η' states in Eq. (15). It can be easily seen below that this is valid only if we set $f_K = f_\pi$ and ignore the electric penguin correction factor ($\xi = 1$).

We would like to point out that whether the experimental value Eq.(1) is inconsistent with Eq. (3) depends crucially on the assumptions above. Looking at the $SU(3)$ relation in general form in Eq. (13), we see that the amplitude of $B^0 \rightarrow K^0 \eta'$ can be enhanced by large r and s' . And in fact, relaxing above assumptions, we can easily have $Br(B^0 \rightarrow K^0 \eta')$ consistent

with data. For example, if we choose $r = 1.1$ and $\theta_p = -10^\circ$, keep the relation $s'/s = -\cos\theta_p/\sin\theta_p$ and take $Br(B^0 \rightarrow K^0\pi^0) = 15 \times 10^{-6}$ and $Br(B^0 \rightarrow K^0\eta) = 0$, Eqs. (12) and (13) give

$$Br(B^0 \rightarrow K^0\eta') = 84 \times 10^{-6}. \quad (16)$$

We insist that $SU(3)$ breaking effects for r , s and s' must be studied before we conclude that experimental data is too large - and that we need new physics to explain the observations. The large value in Eq. (16) is due to the fact that constraints: $Br(B^0 \rightarrow K^0\eta) = 0$ and $s'/s = -\cos\theta_p/\sin\theta_p$ leads to large s' .

In our pQCD approach, we evaluate these $SU(3)$ breaking parameters as well as the branching ratios $Br(\bar{B}^0 \rightarrow \bar{K}^0\eta')$ and $Br(\bar{B}^0 \rightarrow \bar{K}^0\eta)$. Now let us explain our calculation for $\bar{B}^0 \rightarrow \bar{K}^0\eta^{(\prime)}$ decay amplitude. (Full calculation will be presented in elsewhere.) The pQCD approach is developed to give more precise theoretical prediction beyond vacuum saturation approximation [12, 13]. In this approach, the amplitude for $B^0 \rightarrow K^0\eta^{(\prime)}$ is given as (see also Fig. 2):

$$\begin{aligned} M(\bar{B}^0 \rightarrow \bar{K}^0\eta^{(\prime)}) = & \frac{M_B^2 G_F}{2\sqrt{2}} \{ V_t f_K \frac{X_{\eta^{(\prime)}}}{\sqrt{2}} F_e^I + V_t f_y Y_{\eta^{(\prime)}} F_e^{II} \\ & + V_t (f_x \frac{X_{\eta^{(\prime)}}}{\sqrt{2}} \xi + f_x \frac{X_{\eta^{(\prime)}}}{\sqrt{2}} + f_y Y_{\eta^{(\prime)}}) F_e^{III} \\ & - V_u f_x \frac{X_{\eta^{(\prime)}}}{\sqrt{2}} F_e^{IV} + V_t f_B \frac{X_{\eta^{(\prime)}}}{\sqrt{2}} F_a^A + V_t f_B Y_{\eta^{(\prime)}} F_a^B \}, \quad (17) \end{aligned}$$

where ξ is a correction factor which represents the difference between electric penguin contributions of $bsu\bar{u}$ and $bsd\bar{d}(bss\bar{s})$ penguin diagrams and $V_q = V_{qs}^* V_{qb}$ ($q = t, u$). The parameter $a_i(\mu)$ in Fig. 2 is defined as $a_2(\mu) = C_1(\mu) + C_2(\mu)/N$, $a_i(\mu) = C_i(\mu) + C_{i+1}(\mu)/N$ for $i = 3, 5, 7, 9$ and $a_i(\mu) = C_i(\mu) + C_{i-1}(\mu)/N$ for $i = 4, 6, 8, 10$, where N is number of the color and $C_i(\mu)$ is Wilson coefficient (we use the same definition of Wilson coefficient as the one in [12]). It is worthwhile pointing out that scale μ in these coefficients is related to the loop integration variable for diagrams shown in Fig. 2. Thus the usual scale dependence problem associated with the factorization assumption is absent in the pQCD approach.

In our notation, F_e^i ($i = I \sim III(IV)$) represents penguin (tree) diagram and F_a^i ($i = A, B$) represents annihilation penguin diagram. The correspondence between Eq. (9) and (17) is as follows:

$$P_d = \frac{M_B^2 G_F}{2\sqrt{2}} V_t (f_K F_e^I + f_B F_a^A) \quad (18)$$

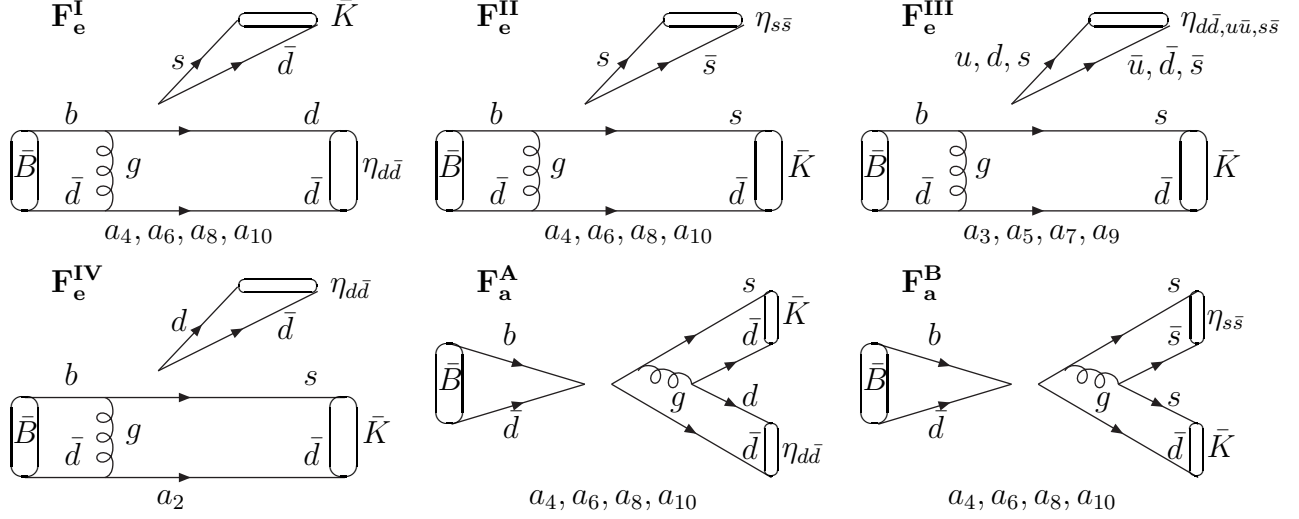


Figure 2: Contributing diagrams.

$$P_s = \frac{M_B^2 G_F}{2\sqrt{2}} V_t (f_y F_e^{II} + f_B F_a^B) \quad (19)$$

$$P^{(\prime)} = \frac{M_B^2 G_F}{2\sqrt{2}} V_t (f_x \frac{X_{\eta^{(\prime)}}}{\sqrt{2}} \xi + f_x \frac{X_{\eta^{(\prime)}}}{\sqrt{2}} + f_y Y_{\eta^{(\prime)}}) F_e^{III}. \quad (20)$$

Note that we also have additional contribution from nonfactorizable diagrams (see, Fig. 3), which is not calculated in vacuum saturation approximation but can be calculable in pQCD approach. However, we found that nonfactorizable contributions to branching ratios for $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ and $\bar{B}^0 \rightarrow \bar{K}^0 \eta$ are less than 10%.

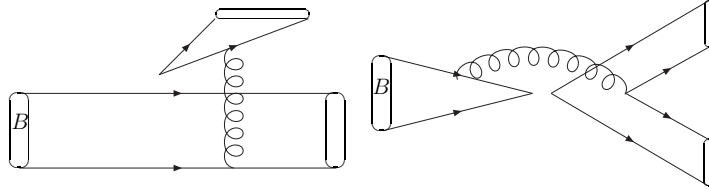


Figure 3: Nonfactorizable contribution

As is mentioned in the introduction, we use new definitions of decay constants in the $\eta - \eta'$ system - the decay constants at the non-anomaly limit [14–16]:

$$if_x p_\mu = \langle 0 | (u \gamma^\mu \gamma_5 \bar{u} + d \gamma^\mu \gamma_5 \bar{d}) / \sqrt{2} | \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \rangle \quad (21)$$

$$if_y p_\mu = < 0 | s \gamma^\mu \gamma_5 \bar{s} | s \bar{s} > . \quad (22)$$

For the value of f_x and f_y , we use the ones that are given in [14] where isospin symmetry is assumed for f_x and $SU(3)$ breaking effect is included for f_y :

$$f_x = f_\pi, \quad f_y = \sqrt{2f_K^2 - f_\pi^2}. \quad (23)$$

These values are translated to the values in the two mixing angle method, which is often used in vacuum saturation approach as:

$$f_8 = 169 \text{ MeV}, \quad f_1 = 151 \text{ MeV}, \quad (24)$$

$$\theta_8 = -28.9^\circ (-18.9^\circ), \quad \theta_1 = -10.1^\circ (-0.1^\circ), \quad (25)$$

where the pseudoscalar mixing angle θ_p is taken as -20° (-10°). The wave function for $d\bar{d}$ components in $\eta^{(\prime)}$ and K meson are given as:

$$\Psi_{\eta_{d\bar{d}}}(P, x, \zeta) \equiv \not{P} \phi_{\eta_{d\bar{d}}}^A(x) + m_0^{\eta_{d\bar{d}}} \phi_{\eta_{d\bar{d}}}^P(x) + \zeta m_0^{\eta_{d\bar{d}}} (\not{p} \not{n} - v \cdot n) \phi_{\eta_{d\bar{d}}}^{\sigma'}(x) \quad (26)$$

$$\Psi_K(P, x, \zeta) \equiv \not{P} \phi_K^A(x) + m_0^K \phi_K^P(x) + \zeta m_0^K (\not{p} \not{n} - v \cdot n) \phi_K^{\sigma'}(x), \quad (27)$$

where P and x are the momentum and the momentum fraction of $\eta_{d\bar{d}}(K)$, respectively. We assumed here that the wave function of $\eta_{d\bar{d}}$ is same as the π wave function. The parameter ζ is either $+1$ or -1 depending on the assignment of the momentum fraction x . $\phi_{\eta_{d\bar{d}}(K)}^A$, $\phi_{\eta_{d\bar{d}}(K)}^P$ and $\phi_{\eta_{d\bar{d}}(K)}^\sigma$ represent the axial vector, pseudoscalar and tensor components of the wave function, respectively, for which we utilize the result from the Light-Cone sum rule [17] including twist-3 contribution:

$$\begin{aligned} \phi_{\eta_{d\bar{d}}}^A(x) &= \frac{3}{\sqrt{2N_c}} f_x x(1-x) [1 + a_2^{\eta_{d\bar{d}}} \frac{3}{2} (5(1-2x)^2 - 1)] + \\ &\quad a_4^{\eta_{d\bar{d}}} \frac{15}{8} (21(1-2x)^4 - 14(1-2x)^2 + 1) \\ \phi_{\eta_{d\bar{d}}}^P(x) &= \frac{1}{2\sqrt{2N_c}} f_x [1 + (30\eta_3 - \frac{5}{2}\rho_{\eta_{d\bar{d}}}^2) \frac{1}{2} (3(1-2x)^2 - 1) + \\ &\quad (-3\eta_3\omega_3 - \frac{27}{20}\rho_{\eta_{d\bar{d}}}^2 - \frac{81}{10}\rho_{\eta_{d\bar{d}}}^2 a_2^{\eta_{d\bar{d}}}) \frac{1}{8} (35(1-2x)^4 - 30(1-2x)^2 + 3)] \\ \phi_{\eta_{d\bar{d}}}^{\sigma'}(x) &= \frac{3}{\sqrt{2N_c}} f_x (1-2x) \\ &\quad [\frac{1}{6} + (5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_{\eta_{d\bar{d}}}^2 - \frac{3}{5}\rho_{\eta_{d\bar{d}}}^2 a_2^{\eta_{d\bar{d}}}) (10x^2 - 10x + 1)] \\ \phi_K^A(x) &= \frac{3}{\sqrt{2N_c}} f_K x(1-x) [1 + 3a_1^K (1-2x) + \frac{3}{2} a_2^K (5(1-2x)^2 - 1)], \end{aligned}$$

where

$$\begin{aligned} a_2^{\eta_{d\bar{d}}} &= 0.44, \quad a_4^{\eta_{d\bar{d}}} = 0.25, \quad a_1^K = 0.20, \quad a_2^K = 0.25, \\ \rho_{\eta_{d\bar{d}}} &= \frac{m_\pi}{m_0^{\eta_{d\bar{d}}}}, \quad \eta_3 = 0.015, \quad \omega_3 = -3. \end{aligned}$$

We assume that the wave function of $u\bar{u}$ is same as the wave function of $d\bar{d}$. For the wave function of the $s\bar{s}$ components, we also use the same form as $d\bar{d}$ but with $m_0^{s\bar{s}}$ and f_y instead of $m_0^{d\bar{d}}$ and f_x , respectively. The pseudoscalar and tensor components of the K wave function are obtained by exchanging parameters of the pseudoscalar and tensor components of $d\bar{d}$ wave function, respectively as follows:

$$f_x \leftrightarrow f_K, \quad \rho_{\eta_{d\bar{d}}} \leftrightarrow \rho_K = \frac{m_K}{m_0^K}, \quad a_{2(4)}^{\eta_{d\bar{d}}} \leftrightarrow a_{1(2)}^K. \quad (28)$$

The parameters m_0^i ($i = \eta_{d\bar{d}(u\bar{u})}, \eta_{s\bar{s}}, K$) are defined as:

$$m_0^{\eta_{d\bar{d}(u\bar{u})}} \equiv m_0^\pi \equiv \frac{m_\pi^2}{(m_u + m_d)}, \quad m_0^{\eta_{s\bar{s}}} \equiv \frac{2M_K^2 - m_\pi^2}{(2m_s)}, \quad m_0^K \equiv \frac{M_K^2}{m_{d(u)} + m_s}. \quad (29)$$

Because there are large ambiguities in quark masses, parameters defined in Eq. (29) introduce considerable theoretical uncertainties. In our analysis, we use constraints for these parameters from analysis of other decay channels. In Ref. [13] branching ratios for $B \rightarrow \pi\pi$ are analysed in pQCD approach. The allowed region for m_0^π is given as $1.1\text{GeV} \leq m_0^\pi \leq 1.9\text{GeV}$. Ref. [12], which studies $B \rightarrow K\pi$ in pQCD, gives the best fit value of $m_s = 140$ MeV.

We saw that the $SU(3)$ breaking effects were included through the decay constants and the wave functions in Eq. (23) and Eq. (29), respectively. In exact $SU(3)$ symmetry limit,

$$f_\pi = f_x = f_y = f_K, \quad (30)$$

$$m_0^{\eta_{d\bar{d}(u\bar{u})}} = m_0^{s\bar{s}} = m_0^K, \quad (31)$$

Eq. (3) is recovered.

Now we show our numerical results. The parameters which are used in our calculation are as follows:

$$\begin{aligned} G_F &= 1.16639 \times 10^{-5} \text{GeV}^{-2}, \quad \Lambda_{\overline{MS}}^{(4)} = 250 \text{ MeV}, \quad \alpha_s(M_Z) = 0.117, \quad \alpha_{em} = 1/129, \\ \tau_{B^0} &= 1.56 \text{ ps}, \quad \lambda = 0.2196, \quad A = 0.819, \quad R_b = \sqrt{\rho^2 + \eta^2} = 0.38, \quad \phi_3 = 90^\circ, \\ M_W &= 80.2 \text{ GeV}, \quad M_B = 5.28 \text{ GeV}, \quad m_t = 170 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}, \\ f_B &= 190 \text{ MeV}, \quad f_K = 160 \text{ MeV}, \quad f_\pi = 130 \text{ MeV} \end{aligned}$$

$f_K \frac{X_{\eta'}}{\sqrt{2}} F_e^I$	-1.57 (-1.93)	$f_K \frac{X_\eta}{\sqrt{2}} F_e^I$	-2.26 (-1.95)
$f_y Y_{\eta'} F_e^{II}$	-4.03 (-3.50)	$f_y Y_\eta F_e^{II}$	2.81 (3.50)
$(f_x X_{\eta'}/\sqrt{2}\xi + f_x X_{\eta'}/\sqrt{2} + f_y Y_{\eta'}) F_e^{III}$	2.16 (2.15)	$(f_x X_\eta/\sqrt{2}\xi + f_x X_\eta/\sqrt{2} + f_y Y_\eta) F_e^{III}$	0.120 (-0.266)
$\frac{V_u}{V_t} f_x \frac{X_{\eta'}}{\sqrt{2}} F_e^{IV}$	$i0.0334 (i0.0411)$	$\frac{V_u}{V_t} \frac{X_\eta}{\sqrt{2}} F_e^{IV}$	$i0.0481 (i0.0417)$
$f_B \frac{X_{\eta'}}{\sqrt{2}} \text{Re} F_a^A$	0.133 (0.163)	$f_B \frac{X_\eta}{\sqrt{2}} \text{Re} F_a^A$	0.191 (0.165)
$f_B \frac{X_{\eta'}}{\sqrt{2}} \text{Im} F_a^A$	0.734 (0.902)	$f_B \frac{X_\eta}{\sqrt{2}} \text{Im} F_a^A$	1.06 (0.915)
$f_B Y_{\eta'} \text{Re} F_a^B$	0.494 (0.428)	$f_B Y_\eta \text{Re} F_a^B$	-0.343 (-0.428)
$f_B Y_{\eta'} \text{Im} F_a^B$	2.00 (1.74)	$f_B Y_\eta \text{Im} F_a^B$	-1.39 (-1.74)

Table 1: The numerical result with the best fit parameter set, $\{m_0^{\eta_{d\bar{d}}(u\bar{u})}, m_s, \omega_B\} = \{1.4\text{GeV}, 140\text{MeV}, 0.4\text{GeV}\}$ for the each term in Eq. (17). The pseudoscalar mixing angle is taken as $\theta_p = -20^\circ(-10^\circ)$. The values are factored out by $10^4 V_t$.

The B meson wave function is given as follows:

$$\phi_B(x) = N_B x^2 (1-x)^2 \exp\left[-\frac{1}{2}\left(\frac{xM_B}{\omega_B}\right)^2 - \frac{\omega_B^2 b^2}{2}\right]$$

$$N_B = 91.7835 \text{ GeV},$$

where ω_B is a free parameter. At first, we show our result with the best fit parameter set which is obtained from analysis of $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$ processes in pQCD approach [12, 13]:

$$\{m_0^{\eta_{d\bar{d}}(u\bar{u})}, m_s, \omega_B\} = \{1.4\text{GeV}, 140\text{MeV}, 0.4\text{GeV}\} \quad (32)$$

Our result of the branching ratio for the above parameter set is

$$Br(\bar{B}^0 \rightarrow \bar{K}^0 \eta') = 18(16) \times 10^{-6} \quad (33)$$

$$Br(\bar{B}^0 \rightarrow \bar{K}^0 \eta) = 0.44(1.9) \times 10^{-6}, \quad (34)$$

for the pseudoscalar mixing angle $\theta_p = -20^\circ(-10^\circ)$. The computed amplitude for the each diagram in Fig. 2 is given in Table 1. With the same set of parameters, we obtain theoretical prediction for $\bar{B}^0 \rightarrow \bar{K}^0 \pi^0$ as $Br(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0) = 9.5 \times 10^{-6}$.

Since we observe large imaginary part for annihilation diagrams we need to rewrite the $SU(3)$ relation in Eqs. (12) and (13) more precisely. Again

ignoring the small amount of tree contribution, we obtain:

$$\text{Re}A(B^0 \rightarrow K^0 \eta) = -(X_\eta + r\sqrt{2}Y_\eta + r\sqrt{2}s)\text{Re}A(B^0 \rightarrow K^0 \pi^0) \quad (35)$$

$$\text{Re}A(B^0 \rightarrow K^0 \eta') = -(X_{\eta'} + r\sqrt{2}Y_{\eta'} + r\sqrt{2}s')\text{Re}A(B^0 \rightarrow K^0 \pi^0) \quad (36)$$

$$\text{Im}A(B^0 \rightarrow K^0 \eta) = -(X_\eta + r'\sqrt{2}Y_\eta)\text{Im}A(B^0 \rightarrow K^0 \pi^0) \quad (37)$$

$$\text{Im}A(B^0 \rightarrow K^0 \eta') = -(X_{\eta'} + r'\sqrt{2}Y_{\eta'})\text{Im}A(B^0 \rightarrow K^0 \pi^0) \quad (38)$$

where

$$r = \frac{f_y F_e^{II} + f_B \text{Re}F_a^B}{f_K F_e^I + f_B \text{Re}F_a^A}, \quad r' = \frac{\text{Im}F_a^B}{\text{Im}F_a^A} \quad (39)$$

$$s' = \frac{(f_x \frac{X_{\eta'}}{\sqrt{2}} \xi + f_x \frac{X_{\eta'}}{\sqrt{2}} + f_y Y_{\eta'}) F_e^{III}}{f_y F_e^{II} + f_B \text{Re}F_a^B}, \quad s = \frac{(f_x \frac{X_\eta}{\sqrt{2}} \xi + f_x \frac{X_\eta}{\sqrt{2}} + f_y Y_\eta) F_e^{III}}{f_y F_e^{II} + f_B \text{Re}F_a^B} \quad (40)$$

Using the values listed in Table 1, the $SU(3)$ breaking effect $r^{(')}$, which is assumed as $r = 1$ in Eq.(3), is calculated as $r = 1.2$ and $r' = 1.3$ and the value of $s^{(')}$ which is proportional to the $SU(3)$ singlet contribution is calculated as $s' = -0.50(-0.50)$ and $s = -0.028(0.059)$ for $\theta_p = -20^\circ(-10^\circ)$. The electric penguin correction factor is obtained as $\xi = 0.543$. To employ the new definition of the decay constant in $\eta - \eta'$ system modifies the relation between s and s' as

$$s'/s = \frac{(f_x \frac{X_{\eta'}}{\sqrt{2}} \xi + f_x \frac{X_{\eta'}}{\sqrt{2}} + f_y Y_{\eta'})}{(f_x \frac{X_\eta}{\sqrt{2}} \xi + f_x \frac{X_\eta}{\sqrt{2}} + f_y Y_\eta)}, \quad (41)$$

which is assumed to be $s'/s = -\cos \theta_p / \sin \theta_p$ in Eq. (3). s' always has minus sign. This is due to the sign difference between F_e^{II} and F_e^{III} , which can be traced back to the relative size of the Wilson coefficients. This effect is also seen in calculations using vacuum saturation approximation [18]. This fact implies that the $SU(3)$ singlet penguin contribution to $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ tends to decrease the branching ratio.

Before we show our numerical results for the different parameter sets of the wave functions, it might be convenient to summarize the trend of the size of the amplitudes for the variation of the parameters, diagram by diagram in Fig. 2. The variable parameters are $m_0^{\eta_{d\bar{d}(u\bar{u})}}$, $m_0^{\eta_{s\bar{s}}}$ and m_0^K which depend on quark masses and ω_B which parameterizes momentum distribution of b quark in a B meson. The amplitudes F_e^I , $\text{Re}F_a^A$ and $\text{Im}F_a^A$ depend on parameters $m_0^{\eta_{d\bar{d}(u\bar{u})}}$ and m_0^K , F_e^{II} , $\text{Re}F_a^B$ and $\text{Im}F_a^B$ depend on parameters $m_0^{\eta_{s\bar{s}}}$ and m_0^K and F_e^{III} depends on m_0^K . All the amplitudes are increased when ω_B is decreased. Note that the branching ratio of $B^0 \rightarrow K^0 \pi^0$ are written in terms of F_e^I , $\text{Re}F_a^A$ and $\text{Im}F_a^A$.

We first discuss the input parameter dependence of s' and s . When we decrease m_s , which increases $m_0^{\eta_{s\bar{s}}}$ and m_0^K , the amplitudes F_e^{II} , $\text{Re}F_a^B$ and F_e^{III} get enhanced simultaneously. When we decrease ω_B , F_e^{II} and F_e^{III} increase while $\text{Re}F_a^B$ remains unchanged. Therefore, as we can see from Eq. (40), s' and s are quite insensitive to the input parameters $m_0^{\eta_{s\bar{s}}}$, m_0^K and ω_B .

Now let us see how we can obtain large $Br(\bar{B}^0 \rightarrow \bar{K}^0 \eta')$. To enhance $Br(\bar{B}^0 \rightarrow \bar{K}^0 \eta')$, we need large r . The value for r is increased for smaller $m_0^{\eta_{d\bar{d}(u\bar{u})}}$, smaller m_s or smaller ω_B . First, let us try $\omega_B = 0.3\text{GeV}$ which is the lower limit considering other two body B decays. For example, we obtain

$$r = 1.2, \quad r' = 1.3, \quad s = -0.032(0.054), \quad s' = -0.49(-0.49) \\ Br(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0) = 21 \times 10^{-6}, \quad Br(\bar{B}^0 \rightarrow \bar{K}^0 \eta') = 34(31) \times 10^{-6},$$

for $\theta_p = -20^\circ(-10^\circ)$. In order to further increase r and thus increase $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$, we need to reduce m_s , however, it ends up also increasing $\bar{B}^0 \rightarrow \bar{K}^0 \pi^0$ branching ratio. This is unacceptable. Therefore, we put back ω_B as 0.4GeV and try to enhance r only by changing $m_0^{\eta_{d\bar{d}(u\bar{u})}}$ and m_s . In fact, the dependence of r on $m_0^{\eta_{d\bar{d}(u\bar{u})}}$ is so weak that even if we take smaller value for $m_0^{\eta_{d\bar{d}(u\bar{u})}}$ the branching ratio for $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ does not become large enough. So, the only possibility is to take a smaller value for m_s . The results evaluated with $\{m_s, \omega\} = \{100\text{MeV}, 0.4\text{GeV}\}$ are as follows:

$$\text{For } m_0^{\eta_{d\bar{d}(u\bar{u})}} = 1.9 \text{ GeV} \\ r = 1.4, \quad r' = 1.3, \quad s = -0.025(0.045), \quad s' = -0.40(-0.40), \quad (42)$$

$$\text{For } m_0^{\eta_{d\bar{d}(u\bar{u})}} = 1.4 \text{ GeV} \\ r = 1.7, \quad r' = 1.5, \quad s = -0.025(0.045), \quad s' = -0.40(-0.40), \quad (43)$$

$$\text{For } m_0^{\eta_{d\bar{d}(u\bar{u})}} = 1.1 \text{ GeV} \\ r = 2.0, \quad r' = 1.6, \quad s = -0.025(0.045), \quad s' = -0.40(-0.40), \quad (44)$$

Obtained branching ratios for $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ and $\bar{B}^0 \rightarrow \bar{K}^0 \pi^0$ are given in Table2. Considering that BELLE reported a smaller branching ratio for $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ and also that experimental value of $\bar{B}^0 \rightarrow \bar{K}^0 \pi^0$ has still large error and can be large, a situation such that $m_0^{\eta_{d\bar{d}(u\bar{u})}} = 1.9 \text{ GeV}$ can not be excluded.

For a comparison to other approaches, we give the obtained values of the form factors for different parameter;

$$F_0^{B \rightarrow \pi}(0) = (0.36, 0.30, 0.26) \quad : \quad m_0^{\eta_{d\bar{d}(u\bar{u})}} = (1.9, 1.4, 1.1)\text{GeV} \\ F_0^{B \rightarrow K}(0) = (0.47, 0.35) \quad : \quad m_s = (100, 140)\text{MeV}$$

where $\omega_b = 0.4\text{GeV}$.

	$Br(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0)$	$Br(\bar{B}^0 \rightarrow \bar{K}^0 \eta')$	$Br(\bar{B}^0 \rightarrow \bar{K}^0 \eta)$
$m_0^{\eta_{d\bar{d}(u\bar{u})}} = 1.9 \text{ GeV}$	21×10^{-6}	$50(45) \times 10^{-6}$	$2.4(7.5) \times 10^{-6}$
$m_0^{\eta_{d\bar{d}(u\bar{u})}} = 1.4 \text{ GeV}$	15×10^{-6}	$45(39) \times 10^{-6}$	$4.6(11) \times 10^{-6}$
$m_0^{\eta_{d\bar{d}(u\bar{u})}} = 1.1 \text{ GeV}$	11×10^{-6}	$41(35) \times 10^{-6}$	$7.0(13) \times 10^{-6}$

Table 2: The numerical results for the branching ratios of $\bar{B}^0 \rightarrow \bar{K}^0 \pi^0$, $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ and $\bar{B}^0 \rightarrow \bar{K}^0 \eta$ with parameters tuned to increase $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$. $\{m_s, \omega\} = \{100\text{MeV}, 0.4\text{GeV}\}$ and different $m_0^{\eta_{d\bar{d}(u\bar{u})}}$. These parameter sets are allowed by the pQCD analysis for $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$ processes. The pseudoscalar mixing angle is taken as $\theta_p = -20^\circ(-10^\circ)$.

In fact, there is another interesting aspect. Looking at Eq. (12), the first and the second term which are the dominant contributions have an opposite sign so that the relative variation of the branching ratio for $B^0 \rightarrow K^0 \eta$ is much larger than that for $B^0 \rightarrow K^0 \eta'$ when r varies. The branching ratio for $B^0 \rightarrow K^0 \eta$, which is considered to be negligible in Eq. (3), is enhanced greatly depending on the parameters. We summarize our numerical results of the branching ratios for $\bar{B}^0 \rightarrow \bar{K}^0 \pi^0$, $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ and $\bar{B}^0 \rightarrow \bar{K}^0 \eta$ in Table 2. We can see a large dependence of the branching ratio for $\bar{B}^0 \rightarrow \bar{K}^0 \eta$ on θ_p . As is mentioned before, $\theta_p = -20^\circ$ is the smallest limit of the allowed region. Hence, our results for $\theta_p = -10^\circ$ indicate that $\bar{B}^0 \rightarrow \bar{K}^0 \eta$ process may be observed soon.

In conclusion, we examined the large branching ratio of $B \rightarrow K\eta'$ process using $SU(3)$ relation in general form, Eqs. (12) and (13). If there is a large $SU(3)$ breaking effect, which means that r is much larger than 1, or there is a large $SU(3)$ singlet penguin contribution, which means s' and s are very large, Eqs. (12) and (13) imply that we would have large branching ratio for $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$. We computed r , s' and s as well as branching ratios for $\bar{B}^0 \rightarrow \bar{K}^0 \pi^0$, $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ and $\bar{B}^0 \rightarrow \bar{K}^0 \eta$ processes in pQCD approach. s' is found to contribute destructively to the other dominant contributions to $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ process. Our numerical result in Table 2 indicates that in a case that the experimental data for $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ will come close to the lower limit of BELLE data, $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ problem can be understood in the standard model. However, in this case, the correlation of the experimental data for $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ to the experimental data for $\bar{B}^0 \rightarrow \bar{K}^0 \pi^0$ and $\bar{B}^0 \rightarrow \bar{K}^0 \eta$ has to be examined carefully. In particular, considering that the relatively large value of θ_p which is close to -10° is favored by recent experiments, we have to keep in mind that $Br(\bar{B}^0 \rightarrow \bar{K}^0 \eta)$ may not be so small. If $Br(\bar{B}^0 \rightarrow \bar{K}^0 \eta')$

remains high at its present value or the combination of the experimental data for $\bar{B}^0 \rightarrow \bar{K}^0 \pi^0$, $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ and $\bar{B}^0 \rightarrow \bar{K}^0 \eta$ deviate from our result in Table 2, it may imply that we need modify our understanding of η' .

Finally, we would like to make a comment on two suggested mechanisms to explain the large branching ratio for $B \rightarrow K \eta'$,

(1) Intrinsic charm contribution:

The Cabbibo allowed $b \rightarrow uc\bar{c}$ process can contribute to $B \rightarrow K \eta'$ if there is intrinsic $c\bar{c}$ content in η' (see Fig. 4(a)) [19, 20].

The amount of the $c\bar{c}$ content in η' which is parameterized by decay constant $f_c^{\eta'}$ as:

$$i f_c^{\eta'} p_\mu = \langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta'(p) \rangle \quad (45)$$

Decay constant $f_c^{\eta'}$ is obtained from the radiative J/ψ decay and two photon process of η' . It was found that the numerical result for the intrinsic charm contribution to $B \rightarrow K \eta'$ decay was very small [14, 15].

(2) The $SU(3)$ singlet contributions:

$B \rightarrow K \eta'$ is produced by fusion of gluons, one gluon from $b \rightarrow sg$ process and another one from spectator. (see Fig. 4(b)) [21, 22, 23].

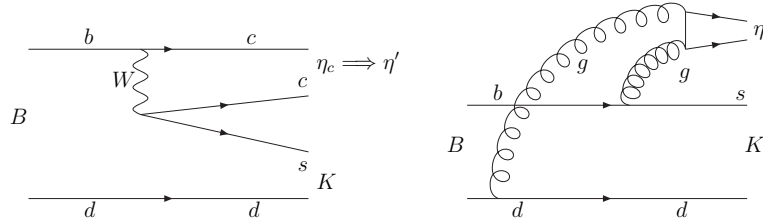


Figure 4: (a) Intrinsic charm contribution, (b) $SU(3)$ singlet contribution

According to the paper [11], there is still a possibility that η' includes at most 26 % of pure gluonic state, gluonium. The contribution of the diagram in which two gluons in Fig. 4(b) are directly attached to gluonium in η' instead of attached to triangle quark loop may be important for $B \rightarrow K \eta'$ decay.

Note added in proof

After this paper was submitted for publication, BELLE and BABAR announced new data:

	$Br(B^0 \rightarrow K^0 \eta')$	$Br(B^+ \rightarrow K^+ \eta')$
BELLE [25]	$(55_{-16}^{+19} \pm 8) \times 10^{-6}$	$(79_{-11}^{+12} \pm 9) \times 10^{-6}$
BABAR [26]	$(42_{-11}^{+13} \pm 4) \times 10^{-6}$	$(70 \pm 8 \pm 5) \times 10^{-6}$

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